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The Energy of Ions Bombarding the Vacuum Chamber Walls

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Abstract

The proton beam circulating in the LHC vacuum chamber may ionise the residual gas. The created ions are accelerated away from the beam and impact the vacuum chamber walls. The value of the ion energy is very important for the estimation of the ion stimulated gas desorption and heat load. The ion energies are estimated for a number of different elements of the LHC vacuum system. The effect of magnetic fields is presented for a first time.

1. INTRODUCTION

Ions, created by ionisation of the residual gas by the proton beam, are repelled from the beam by its positive space charge depending on the beam parameters (bunch spacing and length, the beam size), the mass of ionised molecules and the position where the ions were created.

O.Gröbner [1], [2] has shown that the ion impact energy in the LHC can reach up to 7.2 keV for H_2^+ and 2.8 keV for CO^+ in the IP region, but it does not exceed 300 eV for H_2^+ and 200 eV for CO^+ in other places.

W. Turner [3] also made the estimation of the ion impact energy in the LHC. He obtained the following result: 13.1 keV for H_2^+ and 3.3 keV for CO^+ in the IP region (for a vacuum chamber with ID=80 mm), 165 eV for H_2^+ and 134 eV for CO^+ in the arc (for a vacuum chamber with ID=46 mm).

These two estimations were made for the then nominal current $I = 0.53$ A in the arcs and $I = 2 \times 0.53$ A at an IP. In both studies the effect of an external magnetic field was not taken into account.

The aim of this present study is to estimate the energy of ions bombarding the vacuum chamber walls of different elements of the LHC and to include the effect of magnetic fields at the relevant locations.

2. ION ENERGY IN THE VACUUM CHAMBER WITHOUT MAGNETIC FIELD

Following previous studies we consider a circular beam with a Gaussian profile. The time-averaged electric field of the beam can be given in SI units by:

$$E = \frac{I}{2pe_0 c}\frac{1-e^{-\left(\frac{r}{s_r}\right)^2}}{r}; \qquad (1)$$

where I is the proton beam current;

e_0 = 8.85·10^{-12} [F/m] is the permitivity of free space;

c is the speed of light in vacuum;

s_r is the rms beam size, $s = \sqrt{be_n / g}$, where for LHC: e_n = 3.75·10^{-6} m·rad, g = 7460.6;

r is the distance from the centre of beam to the ion.

In the estimation with a continuos (unbunched) beam the ions arrive at the vacuum chamber wall with a kinetic energy equal to the difference in potential between the point of the ionisation and the wall:

$$W(r_0) = \int_a^R E(r)dr; \qquad (2)$$

where a is the radial position where the molecule was ionised and R is the internal radius of the vacuum chamber. The probability of ionisation $r(a)$ for the residual gas molecules is proportional to the Gaussian distribution and the initial radial position a:

$$r(a) \propto 2pre^{-\left(\frac{a}{s_r}\right)^2}; \qquad (3)$$

then the numerical integrating of equation (2) with K different initial radial positions, a_k (for example: $a_k = 3s_r / k$, $k = 1,2...K$), gives the average value of ion energies:

$$< W >= \sum_{k=1}^{K} w_k W(r_k) ; \tag{4}$$

where w_k is weight of $W(a_k)$: $w_k = \frac{r(a_k)}{\sum_{j=1}^{N} r(a_j)}$.

The estimation described above does not take into account the effect of a bunched beam. In the LHC, the bunch length is $\mathbf{t}$ = 0.257 ns and the bunch spacing is T = 24.95 ns. Hence, the peak of the electric field is about 100 times higher.

$$E_{peak} = \frac{I}{2pe_0 c} \frac{1 - e^{-\left(\frac{r}{s_r}\right)^2}}{r} \frac{T}{t} ; \tag{5}$$

A newly born ion is accelerated by the peak electric field during the bunch passage and then it drifts with a constant velocity until the next bunch arrives. A estimation of its final velocity can be obtained by numerical integration. The iteration formulae for ion velocity and the radial position in the presence of a bunch are:

$$\begin{cases} v_n = v_{n-1} + E_{peak} \dfrac{q}{m} \cdot \Delta t; \\ r_n = r_{n-1} + v_n \cdot \Delta t; \end{cases} \tag{6}$$

where $\Delta t = t / N$ is the time interval, $n = 1,2...N$. The time interval should be chosen small enough so as not to influence the final result. This requirement was found to be satisfactory for N = 1000. The ion drift between two bunches is described as:

$$\begin{cases} v_d = v_N ; \\ r_d = r_N + v_N T. \end{cases} \tag{7}$$

Since the ion can be born at a different radial positions and anywhere along the length of the bunch: in the head, in the middle part or at the tail, the position along the bunch can be described in terms of time, i.e. the duration of acceleration of the ion by the first bunch: $t_1 = \frac{m}{M}t$, $m = 1,2...M$. The formula for the average ion is:

$$<W>= \frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{K} w_k W_k\left(a_k, \frac{m}{M}t \right). \qquad (8)$$

The ion energy is presented in Table 1 for both the present nominal (0.56 A) and ultimate (0.85 A) circulating current for two types of ions, H_2^+ and CO^+. Two beams are present in the same vacuum chamber from the IP to the dipole. Two values are shown in Table 1 in each case: average and maximum (in brackets).

The results of such estimations depend on variation of the beam *b*-function (see Fig. 1 and Fig. 2), the beam current (see Fig. 3) and weakly on the vacuum chamber cross-section (see Fig. 4 and Fig. 5).

The ion energy is not uniform along the vacuum chamber containing both beams; it depends on whether or not the beams arrive at a fixed cross-section simultaneously or with a time delay. The highest average ion energy is reached in the places where the beams arrive simultaneously, the lowest one then the beams arrive in anti-phase. These two extreme cases are presented in Table 1 for the Inner Triplet and dipole D1:

- Case A, when two beams simultaneously arrive to study the cross-section; mathematically the beam current was taken twice higher them in the arc;
- Case B, when two beams arrive in anti-phase, i.e. mathematically the bunch spacing was taken twice less: $T_{1/2} = T/2 = 12.48$ ns.

The beam separation in that region was not taken into account.

Table 1: The energy of ions bombarding the vacuum chamber walls

Element	ID, [mm]	β_{min}, β_{max}, [m]	$<s_r>$, [mm]	W, [eV]			
				at I=0.85A		at I=0.56A	
				H_2^+	CO^+	H_2^+	CO^+
IP1	50-80	0.5	0.022	13600(26500)	2700 (7850)	7500 (15500)	1300 (3750)
Inner triplet							
Q1	46	1060	1.17				
case A		2270		312 (345)	300 (337)	204 (232)	197 (221)
case B				151 (170)	148 (166)	99 (111)	98 (109)
Q2	60	1400	1.58				
case A		4670		304 (345)	295 (332)	199 (225)	194 (218)
case B				148 (167)	146 (164)	97 (109)	96 (108)
Q3	60	2035	1.78				
case A		4710		292 (341)	283 (320)	191 (218)	186 (210)
case B				142 (161)	140 (158)	93 (105)	92 (104)
Dipoles							
D1	130×	1930	1.65				
case A	×55	3520		300 (341)	290 (327)	197 (222)	192 (216)
case B				147 (165)	145 (162)	95 (107)	94(106)
D2	73	470 1650	0.99	184 (204)	180 (198)	120 (133)	119 (131)
Outer triplet							
Q4	50	343 1650	0.88	172 (192)	168 (186)	113 (126)	110 (122)
Q5	50	180 677	0.59	193 (219)	188 (206)	126 (141)	123 (135)
Q6	50	6 395	0.22	275 (375)	238 (258)	170 (252)	157 (169)
Q7	50	59 178	0.29	240 (353)	224 (242)	153 (198)	147 (159)

Element	ID, [mm]	$\beta_{\min}$, $\beta_{\max}$, [m]	$<s_r>$, [mm]	W, [eV]			
				at I=0.85A		at I=0.56A	
				H_2^+	CO^+	H_2^+	CO^+
Arc	46	30 180	0.28	241 (355)	225 (244)	152 (199)	148 (160)

3. ION ENERGY IN THE VACUUM CHAMBER WITH A MAGNETIC FIELD

Along most of the length of the LHC the vacuum chambers are inside magnetic elements: dipoles, quadrupoles, or solenoids... The magnetic field will bend the ion, accelerated by the electric field, and the energy of ion bombarding in the vacuum chamber wall can then differ from the estimation made in section 2 above.

The iteration formulas for the ion velocity and the radial position in magnetic field $\vec{B} = \left(B_x, B_y, B_z\right)$ are

$$\begin{cases} \vec{v}_n = \vec{v}_{n-1} + \dfrac{q}{m}\left(\vec{E} + \vec{v}_{n-1} \times \vec{B}\right) \cdot \Delta t; \\ \vec{r}_n = \vec{r}_{n-1} + \vec{v}_n \cdot \Delta t; \end{cases} \tag{9}$$

where $\vec{E} = \vec{E}_{peak}$ during the bunch passage and $\vec{E} = 0$, in another case.

Three cases were studied for vacuum chamber in:

(1) the dipole magnetic field (Arcs and D1 to D4);
(2) the quadrupole magnetic field (Q1–Q7);
(3) solenoid magnetic field (detectors).

3.1. Vacuum chamber in a dipole magnetic field

Along most of the length of the LHC the vacuum chambers are inside the dipoles in the arcs and the separation dipoles D1–D4 in Long Striaght Sections. The dipole magnetic field strength in the arc is nominally B = 8.4 T. The magnetic field strength in the separation dipoles are B = 1.4 T or B = 3.5 T.

The equation (9) for a dipole magnetic field $\vec{B} = (0, B, 0)$ can be written in a more detailed form such as:

$$\begin{cases} vx_n = vx_{n-1} + \dfrac{q}{m}\left(E \cos a - vz_{n-1}B\right) \cdot \Delta t; \\ vy_n = vy_{n-1} + \dfrac{q}{m} E \sin a \cdot \Delta t; \\ vz_n = vz_{n-1} + \dfrac{q}{m} vx_{n-1} B \cdot \Delta t; \\ x_n = x_{n-1} + vx_n \cdot \Delta t; \\ y_n = y_{n-1} + vy_n \cdot \Delta t; \\ z_n = z_{n-1} + vz_n \cdot \Delta t. \end{cases} \tag{10}$$

The results of estimations of ion energies are presented in Table 2 as a ratio between the average energy with and without taking into account the dipole magnetic field. The H_2^+ energy is higher by factor of 1.05 to 1.15 in the presence of the dipole magnetic field while CO^+ ions have the same energy in both cases.

Hence, in the arcs the average impact energy of ions at maximal beam current does not exceed the value of about 300 eV for H_2^+ and 225 eV for CO^+. The ions will bombard two ~2 mm strips (top and bottom) along a vacuum chamber in a dipole. The incident angle varies between normal and very greasing angles.

In the separation dipoles D1–D4 the effect of a dipole magnetic field is only an increase the average impact energy of H_2^+ less then 10% and does not change the average impact energy of CO^+. H_2^+ and CO^+ ions will bombard two ~4-mm strips (top and bottom) along a vacuum chamber.

3.2. Vacuum chamber in a quadrupole magnetic field

From the point of view of vacuum stability, the vacuum chambers inside the quadrupoles are the most critical elements in the LHC. The quadrupole magnetic field can be described as:

$$\begin{cases} B_x = Gr \sin a; \\ B_y = Gr \cos a; \end{cases} \tag{11}$$

where G is the gradient of the quadrupole magnetic field, α is the angle of radius-vector $\vec{r}$: $x = r\cos\alpha$, $y = r\sin\alpha$.

The equation (9) can be re-written in a more detailed form in this case such as:

$$\begin{cases} vx_n = vx_{n-1} + \frac{q}{m}\left(E - vz_{n-1}B\right)\cos\alpha \cdot \Delta t; \\ vy_n = vy_{n-1} + \frac{q}{m}\left(E + vz_{n-1}B\right)\sin\alpha \cdot \Delta t; \\ vz_n = vz_{n-1} + \frac{q}{m}\left(vx_{n-1}B\cos\alpha - vy_{n-1}B\sin\alpha\right)\cdot \Delta t; \\ x_n = x_{n-1} + vx_n \cdot \Delta t; \\ y_n = y_{n-1} + vy_n \cdot \Delta t; \\ z_n = z_{n-1} + vz_n \cdot \Delta t. \end{cases} \tag{12}$$

The estimations of the ion energy in quadrupoles were made for the maximum gradient of the quadrupole magnetic field *G* = 240 T/m. The results of estimations of ion energies are presented in Table 2 as a ratio between the average energy with and without taking into account the quadrupole magnetic field. As one can see, the H_2^+ energy is higher by 1.3 to 1.7 times in the presence of the quadrupole magnetic field while CO^+ ions have practically the same energy in both cases. Hence, in the quadrupoles the average impact energy of ions at ultimate beam current does not exceed the value of about 500 eV for H_2^+ and 300 eV for CO^+.

Table 2. A ratio between average impact energy with and without a magnetic field.

	H_2^+	CO^+
Arc dipole	1.15	1.00
Arc quadrupole	1.52	
D1–D4	<1.10	1.00
Q1	1.26	1.00–1.02
Q2	1.73	
Q3	1.53	
Q4	1.49	
Q5	1.49	
Q6	1.50	
Q7	1.52	

Figures 6 to 9 show the H_2^+ and CO^+ trajectories with the initial angle of the radius-vector between 0° and 45° in the vacuum chamber of Q3 and Q5. The axes X and Y correspond to the vacuum chamber cross-section and Z is the longitudinal axis. One can see that the ion migration along the vacuum chamber does not exceed the diameter of the vacuum chamber. The ions will bombard four ~4-mm strips along a vacuum chamber in a quadrupole, i.e. about 10% of vacuum chamber surface.

3.3. Vacuum chamber in a solenoid magnetic field

A solenoid field is used in the inner detectors of ATLAS (B = 2 T) and CMS (B = 4 T). The gas density and a vacuum stability of these elements are very important parameters, which depend also on energy of ions bombarding the walls of vacuum chamber.

The iteration formula (10) for the ion velocity and the radial position in a solenoid magnetic field can be re-written in a that case as:

$$\begin{cases} vx_n = vx_{n-1} + \frac{q}{m}\left(E\cos a + vy_{n-1}B\right)\cdot\Delta t; \\ vy_n = vy_{n-1} + \frac{q}{m}\left(E\sin a - vx_{n-1}B\right)\cdot\Delta t; \\ vz_n = vz_0; \\ x_n = x_{n-1} + vx_n\cdot\Delta t; \\ y_n = y_{n-1} + vy_n\cdot\Delta t; \\ z_n = z_0 + vz_0\cdot\Delta t\cdot n. \end{cases} \qquad (13)$$

The estimation of the ion energy in solenoid field was made for the magnetic field B = 2 T and B = 4 T in a vacuum chamber with diameter of 58 mm.

The computing was stopped when either the ion reaches the wall or after 1000 passed bunches. This means that the ion having an initial energy of about 1 eV can move longitudinally no more than 0.25 m, i.e. the ion remains practically with the same co-ordinate z.

Few examples of the ion trajectories near the IP are shown in Figures 10–13. The dependence of ion energy on the number of passed bunches is presented in Figures 14–17.

Table 3. Few examples for the ion energies in a solenoid field near IP.

I, [A]	*b* = 0.5 m			*b* = 2.5 m			*b* = 10 m		
	E, [keV]		Bunches	E, [keV]		Bunches	E, [keV]		Bunches
	H_2^+	CO^+		H_2^+	CO^+		H_2^+	CO^+	
B = 2 Tesla									
0.85	25	7.7	1:5	26	2.1	460:11	30	1.9	720:205
0.56	23	3.6	200:8	26	1.5	>1000:53	9.1	1.6	>1000:400
0.4	34	2.3	997^10	22	1.7	>1000:820	6.4	0.93	>1000
0.3	16	2.1	>1000:440	5.2	0.92	>1000	1.5	1.1	>1000
0.2	12	1.5	>1000:400	9.2	1.5	>1000:300	3.3	0.45	>1000
0.1	4.8	1.2	>1000	4.1	1.3	>1000	4.1	0.49	>1000
B = 4 Tesla									
0.85	156	7.7	332:6	216	5.9	597:250	238	0.7	724:>1000
0.56	188	6.6	470:260	236	1.7	712:>1000	257	1.9	855:>1000
0.4	206	3.4	558:>1000	247	0.84	782:>1000	269	2.9	935:>1000
0.3	254	3.1	811:>1000	263	0.75	898:>1000	264	1.0	893:>1000
0.2	267	5.5	925:>1000	230	0.2	>1000	147	0.15	>1000
0.1	174	1.2	>1000	93	0.4	>1000	89	0.03	>1000

The estimation of ion energies in a solenoid field, presented in Table 3, is made for *b* = 0.5 m (IP), *b* = 2.5 m and *b* = 10 m for ions born at $r = s_r$. The data presented in Table 3 are not average values, as those presented in Table 1, and should only be taken as examples. The column 'bunches' shows how much bunches passed the ion to reach a wall. The indication '>1000' shows that the ion does not reach a wall after 1000 passed bunches. The indicated energy is for 1000 bunches but could be larger.

A more precise estimation of the ion energies would require further study and additional computing time, however a simple estimation of energy range is made below. The ions bent in the solenoid magnetic field and may reach a vacuum chamber wall if the bending radius is larger then half radius of vacuum chamber otherwise the ion will return to the centre of vacuum chamber. As the ions are most strongly accelerated or decelerated close to the beam (inside the beam with size of about s_r) they will circulate between the centre and the wall of the vacuum chamber until the ion gains sufficient energy that its bending radius will be larger than half radius of vacuum chamber. In a stronger magnetic field or larger *b* the ion circulating around a beam, its energy and bending radius increase slowly. The bending radius r of the ion depends on its velocity v as:

$$r = \frac{mv}{qB}. \tag{14}$$

The period T_{ion} is:

$$T_{ion} = \frac{2\pi r}{v} = \frac{2\pi m}{qB}; \tag{15}$$

It can be compared with the bunch spacing is T (see Table 4).

Table 4. The ion's period in a solenoid magnetic field.

	B = 2 T		B = 4 T	
	H_2^+	CO^+	H_2^+	CO^+
T_{ion}, [ns]	65.1	912	32.6	456
T_{ion}/T	2.6	36.5	1.3	18.3

The minimal energy of ions reaching a wall in a solenoid magnetic field can be estimated as:

$$E_1 = \frac{(qBr)^2}{2m} \approx \frac{(qBR)^2}{8m} \tag{16}$$

The minimal impact energy can be estimated as a maximum of two values E_1 and is the energy, estimated in case without magnetic field, E_0:

$$E_{\min} = \max\{E_0, E_1\} \tag{17}$$

The maximal impact energy can be estimated as a maximum of two values.

$$E_{\max} = \max\{E_2, E_3\} \tag{18}$$

First value is the energy E_2 that ion will get if it arrives to about the centre of a vacuum chamber simultaneously with a bunch. This energy can be estimated with numerical integration. The second one comes from maximal possible bending radius in a vacuum chamber, which is about the same as a radius of vacuum chamber.

$$E_3 \le \frac{(qBR)^2}{2m} \tag{19}$$

In general, the case when $E_0 > E_{\max}$ means that magnetic field has no effect on the ion energy. The results of estimation of the energy range for ions in a solenoid magnetic field for a vacuum chamber with R = 29 mm are presented in Table 5.

Table 5. The minimal energy of ions reaching a wall in a solenoid magnetic field.

	B = 2 T		B = 4 T	
	H_2^+	CO^+	H_2^+	CO^+
E_0, [keV]	13.6	2.7	13.6	2.7
E_1, [keV]	20	1.45	80	5.8
E_2, [keV]	36	12	100	12
E_3, [keV]	80	5.8	325	23
$E_{\min}$, [keV]	**20**	**2.7**	**80**	**5.8**
$E_{\max}$, [keV]	**80**	**12**	**325**	**23**
$E¢_{\max}$, [keV]	**~35**	**~8**	**~270**	**~8**

As one can see in Table 3, the number of passed bunches until the ion reaches a wall is larger for a lower beam current. In many cases at I < 0.3–0.4 A the ions does not reach the vacuum chamber walls even after 1000 bunches. The maximal energies in Table 3 lie between E_2 and E_3 but 30–50% lower then $E_{\max}$ in Table 5, the row $E¢_{\max}$ in Table 5 summaries the approximate values of maximal energy from Table 3.

It can be also very important the ions will bombard a wall at greasing incident angles, which can increase the desorption yield.

4. CONCLUSIONS

1. The estimation of the energy of ion bombarding the LHC vacuum chamber without magnetic field sows following:

- The energy of ion bombarding the LHC vacuum chamber wall mostly depends on the beam current and the *b*-function. There is a very weak dependence on a vacuum chamber radius in the range ID = 40 mm to ID = 100 mm.
- The average energy of an ion at maximal beam current is 13.6 keV for H_2^+ and 2.7 keV for CO^+ in IP. In other places between Q1 and Q7 it does not exceed the value of about 300 eV for H_2^+ and CO^+. The ion energy in the arc is 240 eV for H_2^+ and 225 keV for CO^+.
- The ions bombard a wall at normal incident angle.

2. In presence of the quadrupole magnetic field, the H_2^+ ion energy increases by 1.3 to 1.7 times, meanwhile the value of the energy for CO^+ ions are rather insensitive to the quadrupole magnetic field. Hence, in the quadrupoles the average energy of ions at maximal beam current does not exceed the value of about 500 eV for H_2 and 300 eV for CO^+. The ions bombard a wall at different incident angles.
3. The average energy of impact ions near IP with a solenoid magnetic field depends on a vacuum chamber size and magnetic field but does not depend on the *b*-function:
   - The impact ion energy in ATLAS lies in range 20 to 35 keV for H_2^+ and 2.7 to 8 keV for CO^+.
   - The impact ion energy in CMS lies in range 80 to 270 keV for H_2^+ and 5.8 to 8 keV for CO^+.
   - It gets much longer for ions to reach a vacuum chamber wall at lower current; the space charge of ions will be larger at low current
   - The ions bombard a wall at greasing incident angles.

References:

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